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Applications of Functions as a fundamental block in Mathematics

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Abstract:

Functions serve as a foundational tool in mathematics, bridging theoretical concepts with real-world applications across various disciplines. This essay explores the pivotal role of functions in algebra, geometry, trigonometry, and calculus, demonstrating their versatility and practicality. In algebra, linear and quadratic functions model relationships and predict outcomes, such as population growth or projectile motion. Geometry utilizes functions to describe transformations and shapes, enabling advancements in computer graphics and robotics. Trigonometric functions are essential for analyzing periodic phenomena, from tidal patterns to celestial events, while calculus employs functions to study rates of change and accumulation, with applications in physics and economics. By translating real-world problems into mathematical frameworks, functions not only enhance theoretical understanding but also drive innovation in technology and science. This analysis underscores the indispensable nature of functions in both academic research and practical problem-solving, highlighting their transdisciplinary significance.

Keywords: functions, algebra, geometry, trigonometry, calculus, applications.

Introduction

Functions are the most fundamental concepts in mathematics which illustrate the relationship between two sets of components in which there is exactly one output (in the codomain) for every input (from the domain). Thus, many people claim the importance of functions in the discipline of mathematics, emphasizing its fundamental roles. However, it is essential to mention that functions are not only theoretical ideas but a foundational mathematical tool with real-world applications in a wide range of domains. For instance, functions are frequently applied to express transformations in geometry, such as translations and rotations to facilitate the display and manipulation of forms in space (Pasko et al., 1995). On the other hand, it is functions that help to describe relationships in algebra, such as linear and quadratic equations (Doorman & Drijvers, 2011). Besides, trigonometric analysis of periodic patterns including sound waves, ocean tides, and even annual oscillations, also requires an understanding of sine and cosine functions (Odhah et al., 2023). What's more, functions are also important in disciplines beyond mathematics to physics and economics from the perspectives of calculus (Habre & Abboud, 2005). Based on the significance of application, this essay will examine the use of functions in algebra, geometry, trigonometry, and calculus to highlight the significance of functions in mathematical theory and applications.

Functions in Algebra

Algebraic functions are important tools for modeling connections between relating variables (Michelsen, 2006). It is algebraic functions provide a foundation for comprehending the relationships between different quantities to forecast results and optimize planning. In this part, linear functions and quadratic functions will be introduced to address the application of functions in algebra.

Linear function is one of the most basic and frequently used functions in algebra, which define straight-line connections as $f(x)=mx+b$. Such functions are applied for proportional relationship analysis as well as trend prediction. For example, linear functions are often used to model population growth of a city. Suppose the population grows of the city is a fixed number, the relationship between time and population can be represented by a linear function. Similarly, linear functions are also used to forecast financial projections such as revenue or expenses in economics. A company can apply a linear function such as $R(x)=mx+b$, where b stands for fixed income and m is the price per unit. Such a simple function could provide a preliminary picture of a company's future development.

Furthermore, modeling scenarios where change happens at a variable pace requires the use of quadratic functions, which are described by the formula $f(x)=ax^2+bx+c$. Understanding acceleration or curvature phenomena and optimizing systems depend on these functions. The trajectory of a bullet is a well-known illustration of a quadratic function to calculate different variables into the situation. In physics, a quadratic function of time t can be used to describe the height h(t) of an item thrown into the air. For example, $h(t)=-4.9t^2+v_0t+h_0$, where v0 is the starting velocity and h_0 is the initial height. This feature aids in estimating the object's maximum height and landing time.

Functions in Geometry

Functions are frequently used in geometry to describe transformations such as translations, rotations, and reflections (Yanik & Flores, 2009). Transformations of a defining real function are described for set-theoretic operations, blending, offsetting and bijective mapping (Pasko et al., 1995). A key example of functions in geometry is the equation of a circle, $(x-h)^2+(y-k)^2= r^2$. In this function, (h, k) is the center while r is the radius, which can describe the size and location of circles that represented as a function of x and y. By using algebraic methods, issues including tangents location can also be solved. Besides, functions can also used to study geometric shapes such as ellipses, parabolas, and hyperbolas that often appears in the real-life.

There is extensive use of geometry functions, such as matrix functions, in the field of computer graphics (Owens et al., 2007). The geometry functions provide the mathematical foundation to create 3D models for transformation description and shape analyzing. In robotics, functions are used to program the movements of autonomous systems. These uses demonstrate that geometric functions are useful instruments that advance technological advancement rather than only being abstract ideas.

Functions in Trigonometry

Trigonometric functions are effective tools involving angles and triangles (Aga, 2024). In the field of construction, the tangent function tan(θ), is often used to calculate heights and distances of the buildings. According to $h=d\times\tan(\theta)$, the height of the building can be measured based on the building's distance and its angle of elevation could help t. Navigators also frequently employ trigonometric calculations to determine the distances. If a navigator knows the height of a lighthouse and the angle of elevation θ to its top as $d=\frac{h}{tan(\theta)}$, they may calculate the distance to the lighthouse and stimulate the arrival time. Similarly, right-angled triangles are examined with the sine and cosine functions figuring out the sizes of angles in structures like roofs and bridges for the stability and safety of constructions.

More importantly, trigonometric functions are necessary to understand periodic occurrence. Trigonometric functions are used in oceanography to mimic tidal patterns. In astronomy, trigonometric functions are applied to predict celestial events such as eclipse timing, planet positions, and moon phases (Edmunds & Lang, 2009). Even the moon's position in the sky can be described thanks to the sine and cosine functions. Thus, trigonometry functions not only fundamental in mathematic discipline but play an important role for humans in space exploration.

Functions in calculus

Calculus, which mostly relies on functions, is essential to comprehending accumulation. While calculus functions are usually recognized as theoretical concepts in mathe-

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matics, calculus functions are practically applied in engineering and economics (Rasmussen et al., 2014). Specifically, calculus can uses functions to examine curve slopes and rates of change. The slope of the tangent line to the curve at a given point x is represented by the derivative of a function *f*(x) at that point. The derivative *f*′(x) has the following definition:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The formula explains the rate of change of f(x) based on the change of x, which provide the conceptual bases to describe velocity and acceleration. For example, in the field of physics, these calculations help to motion in real-life examples, from the high limit of the space rocket to the stop distance of a racing car. Thus, the role of functions in calculus are noticeable.

What's more, calculus also used to study accumulation through integrals. With the formula of $\int_a^b f(x)dx$, the area under the curve of a *f*(x) from a to b is represented by the integral of *f*(x) across the interval [a,b], which aids in system optimization. Based on that, integrals can be used in economics to calculate important issues such as total profit, which could provide insights into financial planning.

Conclusion

Functions' basic significance in mathematics is demonstrated by their application in algebra, geometry, trigonometry, and calculus. From simple linear functions to exponential functions, it is clear that studying functions provides a wide range of research options. Practical phenomena can be translated into mathematical notions and vice versa by using functions as the language through which we explain and understand the universe. Functions are essential for both fixing current problems and developing new concepts. Based on that, functions strongly help to expand human knowledge including artificial intelligence and space exploration. Thus, learning functions is essential for building comprehension of advanced mathematics. What's more, it is important to point out that leaning functions as an essential part in mathematic is not only about mastering formulas but realizing the application of these functions in real-time situations. By learning the applications of functions, it could shape the knowledge beyond mathematic and develop a more comprehensive understanding of the world.

In conclusion, transdisciplinary problem-solving is based on functions. They are an essential tool for adopting a quantitative perspective on the world. Function analysis is essential to the development of research and technologies that will impact the future of our planet.

References

Aga, T. F. (2024, February 25). Finding the Values of Trigonometric Function without the Use of Scientific Calculators: The Case of Tangent Function. https://www.ajol.info/index.php/ejssd/article/view/265683

Doorman, M., & Drijvers, P. (2011). "Algebra in function". In Secondary Algebra Education. Leiden, The Netherlands: Brill.

Habre, S., & Abboud, M. (2005). Students' conceptual understanding of a function and its derivative in an experimental calculus course. The Journal of Mathematical Behavior, 25(1), 57–72. https://doi.org/10.1016/j.jmathb.2005.11.004

Michelsen, C. (2006). Functions: a modelling tool in mathematics and science. ZDM, 38(3), 269–280. https://doi.org/10.1007/bf02652810

Odhah, O. H., Alshanbari, H. M., Ahmad, Z., Khan, F., & El-Bagoury, A. a. H. (2023). A Novel Probabilistic Approach Based on Trigonometric Function: Model, Theory with Practical Applications. Symmetry, 15(8), 1528. https://doi.org/10.3390/sym15081528

Owens, J. D., Luebke, D., Govindaraju, N., Harris, M., Krüger, J., Lefohn, A. E., & Purcell, T. J. (2007). A survey of General-Purpose Computation on graphics hardware. Computer Graphics Forum, 26(1), 80–113. https://doi.org/10.1111/j.1467-8659.2007.01012.x

Pasko, A., Adzhiev, V., Sourin, A., & Savchenko, V. (1995). Function representation in geometric modeling: concepts, implementation and applications. The Visual Computer, 11(8), 429–446. https://doi.org/10.1007/bf02464333

Rasmussen, C., Marrongelle, K., & Borba, M. C. (2014). Research on calculus: what do we know and where do we need to go? ZDM, 46(4), 507–515. https://doi.org/10.1007/s11858-014-0615-x

Yanik, H. B., & Flores, A. (2009). Understanding rigid geometric transformations: Jeff's learning path for translation. The Journal of Mathematical Behavior, 28(1), 41–57. https://doi.org/10.1016/j.jmathb.2009.04.003