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Unveiling Temperature of the Cosmic Microwave Background Radiation Via Plank's Formula

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Abstract:

From the late 19th century to the early 20th century, blackbody radiation theory lies at the intersection of quantum mechanics and thermodynamics, with its formulation revealing the quantum nature of energy. It resolves the contraction of ultraviolet catastrophe and establishes the functional relationship between the radiation spectrum and temperature. This paper systematically deduces Planck's blackbody radiation law and its inferences, and uses the blackbody radiation formula to invert the cosmic microwave background radiation. Combining theoretical derivation and the extraction of references and observational data, data fitting is achieved through the inversion of multi-spectrum blackbody spectral distribution. The applicability of Wien's displacement law was verified at the theoretical level, and the cosmic microwave background radiation was derived. The blackbody radiation formula and the cosmic microwave background radiation provide benchmarks for cosmological models, supporting the calculation of various parameters and critical temperatures. In the future, perhaps more energy release mechanisms of the early universe can be explored.

Keywords: Blackbody radiation; Planck's blackbody radiation law; Wien's displacement law; Cosmic microwave background radiation.

1. Introduction

Quantum mechanics, a discipline that has profoundly reshaped people's understanding of the microscopic world, marks a significant departure from classical physics through its foundational concept: energy quantization. Unlike the continuous view of energy in classical mechanics, quantum theory postulates that energy exists in discrete units, or "quanta." This revolutionary idea was first introduced by Max Planck in 1900 during his investigation into blackbody radiation, which resolved the ultraviolet catastrophe—a theoretical problem that classical physics could not explain [1,2]. Planck's hypothesis not only accounted for experimental results but also laid the foundation for the entire framework of quantum theory. Subsequent developments further enriched this framework. The notion of wave-particle duality, proposed by Louis de Broglie and later experimentally confirmed, demonstrated that microscopic particles, such as electrons, exhibit both wave-like and particle-like characteristics. This dual behavior has since become a cornerstone of quantum theory, fundamentally altering how people describe matter and radiation. Another pivotal concept is quantum entanglement, which describes the intrinsic connection between particles such that the state of one instantly influences the state of another, irrespective of spatial separation. Once regarded as paradoxical by Einstein, this nonlocal phenomenon has since been experimentally validated and is now utilized in advanced applications such as quantum cryptography, ensuring secure communication.

Quantum theory also introduced the concept of quantum phase transitions, which occur in certain systems under extreme conditions—not due to thermal factors but due to quantum fluctuations. These transitions are critical for the study of low-temperature condensed matter systems, such as superconductors. One of the most remarkable manifestations of quantum mechanics is the Bose-Einstein Condensate (BEC), a state of matter formed when bosons are cooled to temperatures near absolute zero. In this state, particles collapse into a single quantum ground state, effectively behaving as one macroscopic quantum entity. Originally predicted by Einstein and Bose, BECs were experimentally realized in 1995 and continue to serve as an important platform for quantum simulation and fundamental physics research.

In terms of practical and theoretical implications, quantum mechanics has proven indispensable. One of its earliest and most impactful applications lies in explaining the blackbody radiation spectrum—particularly the failure of classical theories to describe energy distribution at high frequencies. Planck's theoretical formulation resolved this issue and provided the foundation for further advancements in quantum statistical mechanics.

In this paper, the authors will explore the theoretical underpinnings of blackbody radiation and its application to the Cosmic Microwave Background (CMB) [3,4]. By deriving Planck's radiation law and applying it to the CMB spectrum, this study aims to provide a clearer understanding of the thermal history of the universe. The implications of this work for modern cosmological models, particularly with respect to parameter estimation and future theoretical developments, will also be discussed. Through this research, the authors seek to bridge the gap between quantum theory and observational cosmology, contributing new insights into the early universe's conditions and offering a deeper perspective on the broader framework of contemporary cosmology.

2. Method and Theory

2.1 Blackbody Radiation and Plank Radiation Law

In classical physics, energy was traditionally regarded as a continuous quantity. However, this assumption failed to account for the observed characteristics of blackbody radiation. Max Planck challenged this notion by proposing that energy is quantized—that is, it can only assume discrete values. Each quantized unit of energy is directly proportional to the frequency of the associated radiation:

$$\mathrm{E}_n = nh\nu, n = 0,1,2,\ldots \quad (1)$$

where h is Planck's constant and n is the quantum number representing the energy level. This equation tells people that the energy in a quantum harmonic oscillator is made of steps, not a smooth range. Under thermal equilibrium, the probability P_n of the system being in a state with energy E_n follows the Boltzmann distribution: $P_n = e^{\frac{-E_n}{kT}} / Z$, where k is the Boltzmann constant, T is the temperature, and Z is the partition function. The partition function can be written as:

$$Z = \sum_{n=0}^{\infty} e^{\frac{-E_n}{kT}} = \sum_{n=0}^{\infty} e^{\frac{-nh\nu}{kT}} \quad (2)$$

This infinite geometric series converges and can be evaluated to yield: $Z = \frac{1}{1-e^{\frac{-h\nu}{kT}}}$. The average energy $\langle E \rangle$ of a quantum harmonic oscillator is given by the weighted average of all possible energy states: $\langle E \rangle = \frac{\sum_{n=0}^{\infty} E_n P_n}{Z}$. Substituting $E_n = nh\nu$ and the value of $P_n = \frac{e^{\frac{-nh\nu}{kT}}}{Z}$, one gets

$$\langle E \rangle = \frac{\sum_{n=0}^{\infty} nh\nu e^{\frac{-nh\nu}{kT}}}{Z} = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1} \quad (3)$$

This equation gives the average energy of a single quantum harmonic oscillator. The number of available modes per unit volume at each frequency is derived from electromagnetic theory. The density of modes is:

$$\rho(\nu) = \frac{8\pi\nu^2}{c^3} \quad (4)$$

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where c is the speed of light. This shows that the number of modes increases with the square of the frequency.

The total energy density $u(\nu,T)$ at frequency ν is the product of the mode density $\rho(\nu)$ and the average energy ?E?:

$$u(\nu,T)=\rho(\nu)\langle E\rangle=\frac{8\pi h\nu^3}{c^3}\cdot\frac{1}{e^{\frac{h\nu}{kT}}-1} \tag{5}$$

This gives the energy density in terms of frequency. To express the energy density in terms of wavelength λ, people use the relation $\nu=\frac{c}{\lambda}$. Thus, the differential $d\nu$ becomes: $d\nu=-\frac{c}{\lambda^2}d\lambda$. Substituting this into the expression for energy density $u(\nu,T)$, one gets the energy density in terms of wavelength λ:

$$M_\lambda(T)=\frac{2\pi hc^2}{\lambda^5}\cdot\frac{1}{e^{\frac{hc}{\lambda kT}}-1} \tag{6}$$

This is the Planck radiation law in terms of wavelength, which also can be expressed in terms of frequency ν as follows:

$$B(\nu,T)=\frac{2h\nu^3}{c^2}\cdot\frac{1}{e^{\frac{h\nu}{kT}}-1} \tag{7}$$

where $B(\nu,T)$ is the spectral radiance per unit frequency [5].

2.2 Derivation of the Planck Radiation Law

At low frequencies (long wavelengths), one has the approximation $\frac{hc}{\lambda kT}\ll 1$, so the authors can expand the exponential term using a Taylor expansion: $e^x\approx 1+x$ when $x\ll 1$. Substituting this into Planck's law, people get the Rayleigh-Jeans law:

$$M_\lambda(T)\approx\frac{2\pi ckT}{\lambda^4} \tag{8}$$

This is valid for low-frequency radiation (long wavelengths). At high frequencies (short wavelengths), one has $\frac{hc}{\lambda kT}\gg 1$, so the authors can approximate the exponential term as $e^x-1\approx e^x$ when $x\gg 1$. Substituting this into Planck's law, one gets Wien's law:

$$M_\lambda(T)\approx\frac{2\pi hc^2}{\lambda^5}e^{\frac{-hc}{\lambda kT}} \tag{9}$$

This approximation is valid for high-frequency radiation (short wavelengths). By integrating Planck's law over all wavelengths, people obtain the total radiated energy per unit area $M(T)=\int_0^\infty M_\lambda(T)d\lambda$. The result of this integration is the Stefan-Boltzmann law:

$$M(T)=\sigma T^4 \tag{10}$$

Where σ is the Stefan-Boltzmann constant, given by $\sigma=\frac{2\pi^5k^4}{15h^3c^2}$. Finally, to determine the wavelength λ_{max} at which radiation is most intense for a given temperature, the authors differentiate Planck's law with respect to λ and set the derivative to zero. This leads to the condition $\frac{dM_\lambda}{d\lambda}=0$. Using a substitution, Let $x=\frac{hc}{\lambda kT}$ convert into $\lambda=\frac{hc}{xkT}$, this simplifies the function by changing variables. In addition, write $M_\lambda(T)$ as a function about x, and calculate: $\lambda^{-5}=(\frac{xkT}{hc})^5$ and $e^{\frac{hc}{\lambda kT}}=e^x$, one can get

$$M(x)=(\frac{xkT}{hc})^5\cdot\frac{1}{e^x-1}. \tag{11}$$

The authors will take the previous constants and analyze only this part of the function, and make $f'(x)=0$ in which $f(x)=\frac{x^5}{e^x-1}$. To find the maximum of this function, one can use the quotient rule and get:

$$f'(x)=\frac{5x^4(e^x-1)-x^5e^x}{(e^x-1)^2}. \tag{12}$$

By setting the numerator to zero, it is found that $5(e^x-1)=xe^x$. Solving numerically, one finds $x\approx 4.9651$ [6], and thus $\lambda_{max}=\frac{hc}{xkT}$ and

$$\lambda_{max}T=\frac{hc}{xk}=b=\frac{hc}{k\cdot 4.9651}\approx 2.897\times10^{-3}m\cdot K \tag{13}$$

This represents the inverse relationship between the peak wavelength of radiation and temperature.

3. Calculation of Cosmic Microwave Background Radiation Temperature

3.1 The Cosmic Microwave Background Radiation

The relevance of blackbody radiation extends into modern cosmology through its direct connection to the CMB radiation. As a relic of the early universe, the CMB provides a snapshot of conditions approximately 380,000 years after the Big Bang, during the epoch of recombination. At this point, the universe had cooled to around 3000 K, allowing electrons and protons to combine and form neutral hydrogen atoms. Prior to this epoch, the universe was a hot, dense plasma in which photons were constantly scattered by free electrons, rendering the universe opaque to electromagnetic radiation. Once the universe expanded and cooled sufficiently, the density of free electrons dropped, allowing photons to travel freely through space for the first time. This event, known as photon decoupling, marked the origin of the CMB, which continues to permeate the universe today.

Recent advancements in CMB research, particularly through the Planck satellite, have provided precise measurements of the CMB spectrum. The latest data, published in the final 2018 dataset, has confirmed that the CMB closely follows the predicted Planck radiation law, with deviations smaller than 1 part in 100,000 [7]. These high-precision measurements have allowed cosmologists to refine key parameters such as the age of the universe, now estimated at 13.78 billion years, and the density of matter and dark energy. Additionally, the detailed analysis of CMB temperature fluctuations and anisotropies has provided compelling evidence supporting the inflationary model of the early universe, confirming that the universe underwent a rapid exponential expansion in the first moments after the Big Bang.

Furthermore, the study of lower-order multipoles, such as the dipole and quadrupole, has shed light on large-scale cosmic structures and the motion of the solar system relative to the CMB's rest frame [8]. Researchers have also explored the potential signals of dark matter in the CMB data, with several models proposing new physics that could explain some of the unexplained anomalies observed in the spectrum. These developments highlight the ongoing need for deeper analysis of the CMB to further people's understanding of the universe's evolution.

The CMB radiation represents the thermal remnants of the early universe which is one of the most profound observational pillars supporting the Big Bang theory. Specifically, from a period approximately 380,000 years after the Big Bang, known as the epoch of recombination. During this phase, the universe had cooled to about 3000 K, allowing electrons and protons to combine and form neutral hydrogen atoms. Prior to this moment, the universe was a hot, dense plasma in which photons were constantly scattered by free electrons , rendering the universe opaque to electromagnetic radiation. As the universe expanded and cooled, the density of free electrons dropped significantly, allowing photons to travel freely through space for the first time—an event known as photon decoupling. These freely streaming photons have continued to travel across the universe largely unimpeded, stretching in wavelength due to the expansion of space itself. This radiation permeates the universe isotropically, with a nearly uniform temperature, and provides critical evidence for the hot, dense initial state of the cosmos.

When expanded in spherical harmonics, the CMB temperature fluctuations can be decomposed into multipoles, The monopole term ($\ell=0$) represents the average temperature of the CMB, a baseline level that is direction-independent and defines the blackbody curve.The dipole term ($\ell=1$) corresponds to a large-scale anisotropy primarily caused by the Doppler effect due to the motion of the solar system relative to the CMB rest frame. It produces a temperature variation of about ±3.3 mK, with one hemisphere appearing slightly warmer and the opposite cooler. Higher-order multipoles ($\ell\geq2$) reveal tiny temperature fluctuations that encode information about the early density variations in the universe, which seeded the formation of galaxies and large-scale structures. The CMB originates from the epoch of recombination, when neutral atoms formed, allowing photons to decouple from matter and travel freely through space. The COBE and FIRAS detected the first thermal spectrum and observed the experimental results are similar to the spectrum of blackbody at T=2.728K.

3.2 Blackbody Radiation

Blackbody radiation is a theoretical construct describing electromagnetic radiation emitted by an idealized object in thermal equilibrium, regardless of frequency or angle of incidence, and re-emits it in a characteristic spectrum solely determined by its temperature. The blackbody nature of the CMB arises from the thermal equilibrium maintained in the primordial plasma prior to photon decoupling. During this era, frequent interactions between photons, electrons, and baryons ensured energy redistribution, imprinting a Planck spectrum. After decoupling, the absence of significant energy injection or absorption mechanisms preserved the spectrum's shape, despite cosmological redshift. The spectral energy distribution of a blackbody is described by Planck's law:

$$P(v)=\frac{2hv^3}{c^2}\frac{1}{e^{\frac{hv}{kT}}-1} \tag{14}$$

where P(v) is the spectral radiance, v is the frequency, T is the temperature, h is Planck's constant, k is Boltzmann's constant, and c is the speed of light. A perfect blackbody spectrum is characterized by its universality dependent solely on temperature and its lack of spectral distortions. Measurements by the FIRAS instrument aboard the COBE

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satellite demonstrated that the CMB spectrum matches the theoretical blackbody spectrum with unprecedented precision. The deviations are less than 1 part in 10^5, making the CMB the most perfect blackbody spectrum ever observed in nature [9].

3.3 Derivation and Data Fitting

To get the energy per unit volume per unit frequency should integrate radiance over solid angles and divide by the speed of light *c* relating energy flow and energy density. This gives people the energy density spectrum,the energy per unit volume per unit frequency.

$$u_v(T) = \frac{4\pi}{c} P(v) = \frac{8\pi h v^3}{c^3} \frac{1}{e^{\frac{hv}{kT}} - 1} \quad (15)$$

Now should get the total energy contained in the radiation field, not just a slice at a specific frequency u(T) by substitution integration.

$$u(T) = \int_0^\infty u_v(T) dv = \int_0^\infty \frac{8\pi h v^3}{c^3} \frac{1}{e^{\frac{hv}{kT}} - 1} dv \quad (16)$$

According to a result from Bose-Einstein statistic. Substituting this yields:

$$u(T) = \frac{8\pi^5 T^4}{15c^3 h^3} T^4 = aT^4 \quad (17)$$

The FIRAS instrument on the COBE satellite measured the CMB spectral energy density across a frequency range of 60–600 GHz. The spectral data closely followed the theoretical Planck distribution, with deviations smaller than 0.005%, affirming the thermal origin of the CMB. FIRAS also provided an integrated energy density of:

$$u = 4.17 \times 10^{-14} J \cdot m^{-3} \quad (18)$$

To estimate the temperature Tof the CMB corresponding to this energy density or by measuring the intensity of the radiation at ultiple frequencies $T^4 \approx 421.1$ [3,4].

The fluctuations in temperature are dominated by random processes. The quantum fluctuations and surges in the early universe, after amplification, approximate the Gaussian model. The observed values of the three frequency points were established as parametric equations for solution, and multiple sets of frequency data were introduced. Since the lower frequency region and the high frequency region have different temperature sensitivities, cross-verification of the differences and distribution patterns of temperature sensitivity in different frequency bands is required.

One can fit the data to the Planck function to determine T. However, to achieve high precision, the full Planck formula must be used. Modern analyses employ parametric fitting to the observed spectrum. For example, the COBE-FIRAS data were modeled by minimizing the residual between observed fluxes and the theoretical curve.

$$x^2 = \sum_{\iota} \frac{\left[B_v^{observed} - B_v(T)\right]^2}{\delta^2} \quad (19)$$

δ is measurement uncertainty where the present measurement uncertainties. The best-fit temperature corresponds to the T that minimizes. This method yielded $T = 2.7255 \pm 0.0006K$, a value corroborated by subsequent missions like the Planck satellite [3,4].

Conclusion

This article focuses on the theoretical basis of blackbody radiation and derives the core expression of Planck's radiation law through quantum statistical mechanics methods. This law describes the pattern of radiation energy density changing with frequency, specifically manifested as a combination of the cubic term of frequency and the exponential function. The study further analyzes the form of this law in the low-frequency and high-frequency limits, namely Rayleigh-Jeans law and Wien approximation, and deduces the inverse relationship between the peak wavelength of blackbody radiation and temperature - Wien displacement law, revealing the essential connection between the microscopic quantum laws and the macroscopic thermal radiation phenomenon. Based on the full-band observational data from the Cosmic Background Explorer and the Planck satellite, this study refers to the data fitting literature, and by minimizing the sum of the squared deviations between the observed values and the theoretical values, finally determines the reference temperature of the cosmic microwave background radiation to be 2.73K. However, this article does not consider the heating effect of Compton scattering during the reionization period of the universe on the blackbody spectrum. And only focus on the unipolar temperature because it characterizes the average energy temperature of the CMB and a baseline level that is direction-independent and defines the blackbody curve ignore the temperature data fitting situation of the dipole.

Authors Contribution
All the authors contributed equally and the names were listed in alphabetical order.

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