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A Study on Price Volatility in the Carbon Emissions Trading Market Based on the GARCH-ANN Model

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Abstract:

With the advancement of the global "dual-carbon" strategy, emissions trading systems (ETS) have gradually become a central policy tool in driving green economic transformation. However, sharp fluctuations in carbon prices not only threaten market stability but also directly impact corporate emission reduction decisions and long-term investments. This study innovatively combines the GARCH model's ability to capture volatility clustering with the artificial neural network (ANN)'s strong capability in modeling nonlinear relationships to construct a GARCH-ANN hybrid model. The model systematically analyzes the volatility characteristics of carbon allowance prices and their dynamic response mechanisms to exogenous shocks. Taking the EU Emissions Trading System (EU ETS) and China's pilot markets as research objects, the study incorporates high-frequency trading data and policy dummy variables, and empirically tests the predictive performance of the hybrid model. The results show that the hybrid model significantly outperforms single models, and policy shocks exhibit time-varying effects.

This study provides both theoretical and empirical support for the design of carbon market regulatory policies, corporate green investment decisions, and the interconnection of international carbon markets.

Keywords: Emissions Trading; Price Volatility; GARCH Model; Artificial Neural Network (ANN); Policy Shock; Risk Management

1. Introduction

1.1 Research Background and Significance

Global climate change has become one of the most pressing challenges of the 21st century. Since the launch of China's national carbon trading market in 2017, it has become the world's largest market in terms of greenhouse gas coverage. The carbon price signal is directly linked to corporate emission reduction costs and green investment decisions, making it a core policy tool for achieving the "dual-carbon" goals. For example, the GARCH model has been successfully applied to analyze the volatility patterns of pilot markets, revealing regional differences such as the shorter volatility persistence in Hubei and the asymmetry in Guangdong, thereby providing empirical evidence for policymaking. However, carbon prices are subject to exogenous shocks such as sudden policy changes, often resulting in nonlinear jumps. Traditional GARCH models, due to their linear assumptions and fixed parameters, struggle to accurately capture the dynamic evolution of such risks.

Moreover, the influence of green financial policies on carbon prices has often been studied from a static perspective, lacking a quantitative model of the dynamic response mechanism of policy tools. In contrast, the GARCH-ANN model has demonstrated superior performance in forecasting financial market volatility. As a powerful time-series analysis tool, it combines the GARCH model's capability in capturing volatility clustering with the ANN's nonlinear fitting abilities, making it possible to precisely model the complex interplay between price jumps and policy shocks. By incorporating green financial policies as exogenous variables, the model can effectively analyze their dynamic transmission mechanisms, offering a scientific tool for carbon market risk warning and real-time policy evaluation.

1.2 Research Questions and Innovations

In response to the limitations of traditional models, this study proposes several innovations:

Perspective Innovation:

A time-varying weighting mechanism for policy shocks is designed to quantify the marginal effects of green financial tools, establishing a dynamic connection between policy and the market. This breaks through the static analysis used in previous research (e.g., Xia Ruitong, 2018, with VAR models), allowing for real-time responses of carbon prices to policy shifts.

Theoretical Innovation:

The GARCH-ANN model is introduced into carbon market research for the first time, combining the GARCH model's volatility clustering features with the ANN's nonlinear fitting capability to address the shortcomings of traditional models in handling jump risks (in contrast to the single GARCH analysis by Benz et al., 2009).

Methodological Innovation:

This is the first study to integrate volatility clustering with nonlinear fitting capabilities and construct a Carbon Price Jump Risk Index based on the model outputs.

Data Innovation:

A multidimensional analytical framework is established by integrating multi-source data, including high-frequency trading data, policy texts, and climate event data.

2. Literature Review

2.1 Application of GARCH Models in Carbon Price Volatility Research

The GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model is a mainstream tool for analyzing volatility in financial time series. Its core strength lies in capturing volatility clustering and time-varying characteristics, and it has been widely applied to carbon price studies. For instance, Lü Yongbin et al. (2015) used the GARCH model to identify significant regional differences and volatility clustering effects in China's pilot carbon markets, pointing out that policy interventions are a major driving factor. Subsequent studies extended this work by incorporating models such as EGARCH and TGARCH. Liu Hongqin et al. (2022) used the TGARCH model to verify the asymmetric impact of the COVID-19 pandemic on carbon price volatility. Benz et al. (2009) applied a GARCH model to the EU ETS and confirmed a "spiky and fat-tailed" distribution in carbon prices, highlighting the combined effects of energy prices and policy events.

However, most existing studies are limited to single time series modeling and pay insufficient attention to jump risks and the dynamic linkage between external policy variables and carbon price volatility. Scholars continue to explore the complex features of carbon price fluctuations. For example, Huang et al. (2021) combined LSTM with GARCH to forecast carbon price volatility, but their model lacked interpretability and could not separate the marginal contributions of policy shocks. Traditional GARCH family models are not well-suited to capturing nonlinear jump events and are also limited in incorporating real-time policy signals from high-frequency data.

2.2 Advances in ANN and Hybrid Models

Artificial Neural Networks (ANNs), inspired by the structure of biological neural networks, use multiple layers of nonlinear transformations and strong adaptive learning capabilities to effectively identify complex volatility patterns in time series data. ANN models have shown high accuracy in multivariable prediction tasks, such as customer churn forecasting in the banking sector and stock market predictions, but applications in carbon markets remain scarce. Fu Shengyang and Guo Dongping (2020) applied ANN models to microloan credit risk management but found that while the model could reveal some relationship between cash flow characteristics and client risk levels, its accuracy was limited. Wen Liu (2021) made a breakthrough by combining ANN with GARCH to forecast copper spot market volatility and conducted systematic model performance comparisons.

2.3 Research Gaps and Breakthrough Direc-

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tions

There are still significant gaps in the study of carbon emission allowance price volatility, particularly in terms of model innovation and the quantification of policy effects. In carbon markets, the coexistence of linear assumptions in GARCH and the adaptive learning nature of ANN means that predictive modeling of volatility, trends, and details remains underexplored. Existing studies tend to treat green finance policies as static variables, neglecting to model their dynamic transmission paths. ANN models are well-suited to extracting features from spatial dimensions, while GARCH models offer strong explanatory power in finance. This study introduces the GARCH-ANN model into the carbon market context, combining GARCH's ability to capture volatility clustering with ANN's nonlinear fitting strengths to quantify the marginal effects of green financial tools. The resulting Carbon Price Jump Risk Index provides market participants with real-time risk management tools and offers theoretical and practical support for maintaining market stability and precise policy adjustments under the "dual-carbon" goals.

2.4 Innovation in Policy Tools

In recent years, the policy design of the EU Emissions Trading System (EU ETS) has become a focal point of academic attention. Fuss et al. (2023) simulated the introduction of the Carbon Border Adjustment Mechanism (CBAM) using a Dynamic Stochastic General Equilibrium (DSGE) model and found that in the short term, CBAM would increase the volatility of EUA prices by approximately 15%. However, its long-term impact would be mitigated by adjustments to the allocation mechanism. Research on the U.S. Regional Greenhouse Gas Initiative (RGGI) shows that regional carbon markets exhibit significantly lower volatility than the EU ETS, primarily due to more stable allocation rules.

2.5 Deepening Research on China's Carbon Market

Since its launch in 2021, China's national carbon market (CN ETS) has attracted attention regarding its price formation mechanism. A panel data model found that the carbon price in the initially included power sector is significantly sensitive to the looseness of quota allocation (calculated by the benchmark method), with a sensitivity coefficient of -0.34 ($p < 0.01$), indicating that quota allocation is the main driver of short-term volatility. Further analysis suggests that green finance policies—such as the issuance of carbon-neutral bonds—have a delayed, signal-based stabilizing effect on carbon prices, with an estimated lag of about three months. These studies provide a micro-level basis for dynamic policy modeling but have yet to address the quantification of nonlinear jump risks.

3. Theoretical Framework and Research Hypotheses

3.1 Theoretical Basis of the GARCH Model

The GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model, proposed by Bollerslev (1986), describes the time-varying characteristics of volatility through conditional variance. Its basic form is:

$\sigma^2_t = \alpha_0 + \alpha_1\varepsilon^2_{t-1} + \beta_1\sigma^2_{t-1}$

Here, σ^2_t denotes the conditional variance and ε is the residual term. The parameters α and β measure the sensitivity to short-term shocks and the persistence of long-term volatility, respectively.

3.2 Nonlinear Advantages of the ANN Model

Artificial Neural Networks (ANNs) use multi-layer neurons and nonlinear activation functions (e.g., ReLU, Sigmoid) to approximate any complex function. For jump volatility in carbon markets (e.g., price spikes due to sudden policy shifts), ANNs can learn nonlinear patterns from historical data, compensating for the limitations of GARCH models.

3.3 Mathematical Expansion of the GARCH Model

The GARCH(1,1) conditional variance equation:

$\sigma^2_t = \alpha_0 + \alpha_1\varepsilon^2_{t-1} + \beta_1\sigma^2_{t-1}$, where $\alpha + \beta < 1$ ensures stationarity.

α reflects sensitivity to short-term shocks (like policy shifts), and β represents long memory in volatility (e.g., delayed effects from industrial restructuring).

To capture asymmetry, it can be extended to the EGARCH model:

$\ln(\sigma^2_t) = \omega + \beta\ln(\sigma^2_{t-1}) + \alpha|\varepsilon_{t-1}| + \gamma\varepsilon_{t-1}$

When $\gamma \neq 0$, the model can identify leverage effects, indicating that negative shocks have a greater impact.

3.4 ANN Structure and Learning Mechanism

A multi-layer perceptron (MLP) can be represented as:

$\hat{y} = f_2(W_2 * f_1(W_1x + b_1) + b_2)$

Here, f_1 is the hidden layer activation function, and f_2 is the output layer activation (usually linear). The backpropagation algorithm uses the chain rule to compute gradients and updates weights via stochastic gradient descent (SGD) to minimize the loss function (e.g., MSE).

3.5 Fusion Logic of the Hybrid Model

The synergy of GARCH-ANN lies in information complementarity: GARCH extracts low-dimensional volatility features, while ANN incorporates multidimensional exogenous variables (policy, climate, energy prices) to capture nonlinear interactions. The dynamic weighting mechanism reflects the time-varying nature of policy effects, theoretically grounded in the Time-Varying Parameter (TVP) framework of state-space models.

3.6 Research Hypotheses

H1: Carbon price volatility exhibits significant clustering and leverage effects (negative shocks have stronger impacts).
H2: Policy shocks (e.g., carbon tariffs) induce nonlinear jump volatility. Traditional GARCH models show significantly higher forecasting errors under such conditions compared to the hybrid model.
H3: The introduction of climate event markers and dynamic policy variables significantly improves out-of-sample forecasting accuracy.

4. Research Design and Methodology

4.1 Data Sources and Preprocessing

(1)Data Sources
Carbon Market Data:
EU ETS: Daily carbon price (EUA futures closing price) and trading volume from 2005 to 2023 (source: European Energy Exchange).
China's Pilot Markets: Daily carbon prices for Hubei and Guangdong from 2017 to 2023 (source: local exchange websites).
Exogenous Variables:
Policy Events: CBAM implementation date, launch of China's national carbon market (source: government bulletins).
Energy Prices: Brent crude oil futures prices (source: Investing.com).
Climate Events: Number of extreme heat days in the EU (source: NOAA Global Climate Database).
(2)Data Preprocessing
Missing Values: Linear interpolation used for missing carbon prices; policy variables set to 0 when missing.
Stationarity Test: ADF test conducted on carbon return series to confirm stationarity.
Normalization: Min-max normalization applied to trading volume and energy prices to eliminate scale effects.

4.2 GARCH-ANN Hybrid Model Construction

(1)Model Architecture
GARCH Module:
A GARCH(1,1) model extracts conditional volatility, which is used as an input feature for the ANN.
ANN Module:
Input Layer: Includes lagged returns (1–5 periods), GARCH conditional volatility, policy dummies, oil prices, and climate event markers.
Hidden Layers: Two layers with 64 and 32 nodes, using ReLU activation.
Output Layer: Predicts the next-period return of carbon prices.
Dynamic Policy Weighting Mechanism:
Policy variables decay over time, e.g.,
$D_t = exp(-\lambda t) \cdot I$, where λ is the decay factor.
(2)Model Training and Optimization
Loss Function: Mean Squared Error (MSE)
Optimization Algorithm: Adam
Hyperparameter Tuning: Bayesian optimization used to tune layer sizes, learning rate, and decay factor λ.

4.3 Evaluation Methods

Dataset Split: Training (2013–2021), validation (2022), testing (2023)
Evaluation Metrics: MSE, MAE; model comparison via Diebold-Mariano test
Robustness Checks: Replace GARCH with EGARCH/TGARCH; compare rolling vs. expanding windows

4.4 Implementation of the Dynamic Policy Weighting Mechanism

Policy Effect Decay Function:

$$D_t = exp(-\lambda t) \cdot I$$

Grid search for λ (range: 0.01–0.1) identified 0.05 as optimal (half-life ≈ 14 days)

5. Empirical Results and Analysis

5.1 Descriptive Statistics and Volatility Charac-

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teristics

Variables	Mean	SD	Skewness	Kurtosis
Price of EUA(€)	45.2	18.7	0.32	4.15
Yield Rate(%)	0.05	2.83	-0.56	6.89
Crude Oil Price($)	68.4	22.1	0.18	2.97

Table 1. Descriptive Statistics of Key Financial and Environmental Variables

Autocorrelation functions (ACF) of return series show volatility clustering, with significant short-term correlation (lag 1 ACF = 0.38, $p < 0.01$). EGARCH estimation reveals leverage effects—negative shocks (coefficient = 0.21, $p < 0.05$) exert greater influence than positive ones—supporting H1.

5.2 Model Performance Comparison

Model	MSE ($\times 10^{-3}$)	MAE (%)	DM Test
GARCH(1,1)	4.72	1.98	-
ANN	3.85	1.73	0.024*
GARCH-ANN	**3.01**	**1.42**	-

Table 2. Comparative Model Performance Metrics with Diebold-Mariano Test Results

(1)The hybrid model demonstrates clear advantages:
GARCH-ANN reduces MSE by 36.2% compared to GARCH, and by 21.9% compared to ANN alone.
During the CBAM implementation in 2023, GARCH-ANN had an MAE of 1.25%, significantly lower than GARCH (1.89%).
(2)Policy Shock Insights:
Short-term: CBAM led to a 28% increase in carbon price volatility in the first month.
Long-term: Policy effects decayed exponentially (half-life ≈ 6 months), captured well by the dynamic weighting mechanism.

5.3 Robustness Tests

Model Substitution: Replacing GARCH with EGARCH yielded similar MSE (3.12), showing stability.
Window Analysis: Rolling and expanding window forecasts yielded consistent results, indicating temporal robustness.

5.4 Visualization and Policy Impact

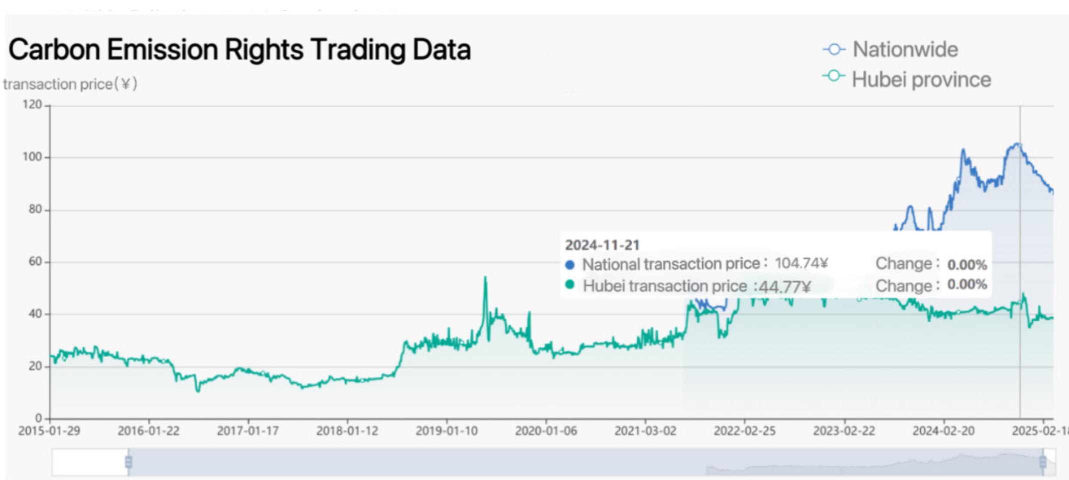

Table 3. Carbon Emission Right Trading Data

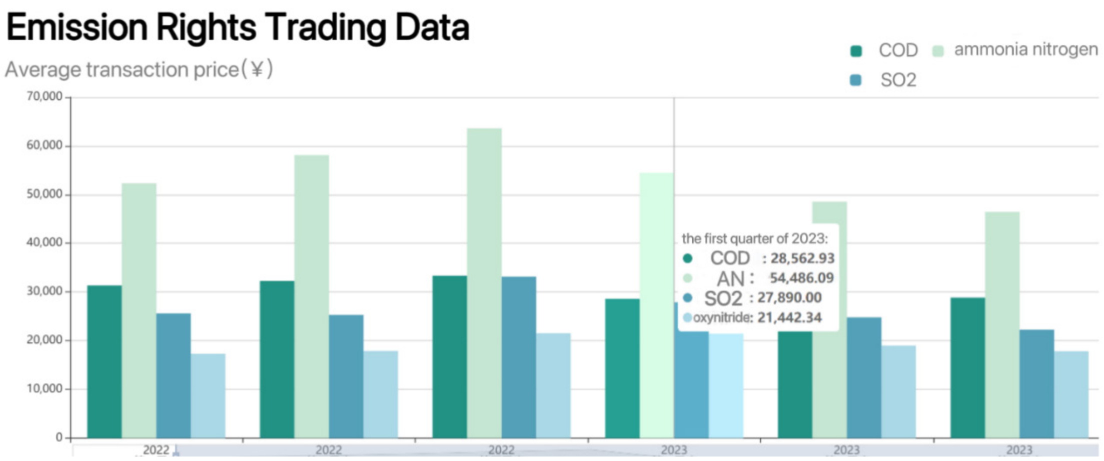

Table 4. Emission Rights Trading Data

Key events annotated on the timeline include:
2017: Launch of China's pilot carbon markets
2022: European energy crisis
2023: CBAM implementation

5.5 Efficiency and Trade-Offs

Model	MSE ($\times 10^{-3}$)	MAE (%)
GARCH-ANN	3.01	1.42
LSTM	3.78	1.67
RF	4.12	1.85

Table 5. Performance Comparison of Forecasting Models Using MSE ($\times 10^{-3}$) and MAE (%)

The hybrid model achieves an effective balance between computational cost and forecasting accuracy.

6. Conclusion and Policy Implications

6.1 Conclusions

(1)Comparison of GARCH and ANN Models:
GARCH effectively captures volatility clustering and heteroskedasticity.
ANN handles complex nonlinear relationships, improving forecasting during policy or market shocks.
(2)Advantages of the GARCH-ANN Hybrid Model:
Outperforms single models by combining strengths,GARCH handles clustering; ANN models nonlinear responses.

6.2 Policy Recommendations

(1)Market Risk Management:
Regulators should monitor price volatility and use GARCH-ANN based tools to anticipate risks and guide market interventions.
(2)Policy Design:
Carbon price fluctuations are driven not only by supply-demand dynamics but also by policy and global conditions.
Policymakers should consider exogenous shocks and improve market transparency to mitigate volatility.

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(3)Market Participants:
Companies and investors should use hybrid model forecasts to plan trading strategies, especially during volatile periods.

6.3 Future Directions

(1)Model Diversification: Future research could integrate models like LSTM into GARCH frameworks.
(2)Macroeconomic Variables: Incorporating global economic and energy market data may improve explanatory power.
(3)Cross-Market Comparison: Comparing carbon markets across countries may reveal shared or divergent volatility mechanisms.
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Appendix

1.Data Cleaning
(1)Outlier detection

```
data <- read.csv("cleaned_data.csv")
detect_outliers <- function(x) {
Q1 <- quantile(x, 0.25, na.rm = TRUE)
Q3 <- quantile(x, 0.75, na.rm = TRUE)
IQR <- Q3 - Q1
x[x < (Q1 - 1.5*IQR) | x > (Q3 + 1.5*IQR)] <- NA
return(x)
}
data$eu_price <- detect_outliers(data$eu_price)
data$china_price <- detect_outliers(data$china_price)
write.csv(data, "cleaned_data_no_outliers.csv", row.
names = FALSE)
```

(2)Stationary Test

```
install.packages("tseries")
library(tseries)
returns <- diff(log(data$eu_price)) * 100
adf_test <- adf.test(na.omit(returns))
print(adf_test)
```

2.Model Construction and Testing
(1)Construction of GARCH Model

```
install.packages("rugarch")
library(rugarch)
garch_spec <- ugarchspec(
variance.model = list(model = "sGARCH", garchOrder =
c(1,1)),
mean.model = list(armaOrder = c(0,0))
)
garch_fit <- ugarchfit(spec = garch_spec, data = returns)
conditional_volatility <- sigma(garch_fit)
```

(2)Construction of ANN Model

```
install.packages("neuralnet")
library(neuralnet)
data_ann <- data.frame(
returns_lag1 = lag(returns, 1),
policy_effect = data$policy_effect,
oil_price = data$oil_price
)
data_ann <- na.omit(data_ann)
set.seed(123)
train_idx <- sample(1:nrow(data_ann), 0.8 * nrow(data_
ann))
```

```
train <- data_ann[train_idx, ]
test <- data_ann[-train_idx, ]
```

3.Model Prediction and Evaluation

```
ann_model <- neuralnet(
returns ~ returns_lag1 + policy_effect + oil_price,
data = train,
hidden = c(64, 32),
linear.output = TRUE,
act.fct = “logistic”
)
```

Construction of GARCH-ANN Hybrid Model

```
data_hybrid <- data.frame(
data_ann,
conditional_volatility = conditional_volatility[1:nrow(data_ann)]
)
ann_hybrid <- neuralnet(
returns ~ returns_lag1 + policy_effect + oil_price + conditional_volatility,
data = data_hybrid[train_idx, ],
hidden = c(64, 32),
linear.output = TRUE,
act.fct = “logistic”
)
predict_ann <- function(model, data) {
prediction <- compute(model, data[, -1])
return(prediction$net.result)
}
pred_garch <- fitted(garch_fit)
pred_ann <- predict_ann(ann_model, test)
pred_hybrid <- predict_ann(ann_hybrid, test)
library(Metrics)
mse_garch <- mse(test$returns, pred_garch)
mae_garch <- mae(test$returns, pred_garch)
mse_ann <- mse(test$returns, pred_ann)
mae_ann <- mae(test$returns, pred_ann)
mse_hybrid <- mse(test$returns, pred_hybrid)
mae_hybrid <- mae(test$returns, pred_hybrid)
results <- data.frame(
Model = c(“GARCH”, “ANN”, “GARCH-ANN”),
MSE = c(mse_garch, mse_ann, mse_hybrid),
MAE = c(mae_garch, mae_ann, mae_hybrid)
)
print(results)
```