

# Modeling Involution: An Evolutionary Game Theory Analysis of Arms-Race Effort Competitions

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## Abstract:

Involution—socially inefficient intensification of individual effort in competitive areas like work and education—arises from individually optimal arms-race dynamics that are collectively suboptimal. To explore the phenomenon, the current study sets up a dynamic model, thinking of involution as an evolutionary game under replicator dynamics. A two-strategy model—High Effort (H) and Low Effort (L)—is set up with payoff externalities reflecting congestion and relative-performance rewards. The model generates a high-effort dominant attractor or an interior evolutionarily stable state, depending on parameters. Closed-form solutions are derived for the selection gradient, interior equilibrium, comparative statics, local stability, and social optimum maximizing average payoff. The analysis connects the mathematics to real examples of schooling, firms, and industries and calculates short-run benefits (peak performance, tournament selection) and major costs (welfare loss, stagnation of innovation, burnout, inequality). Policy and institutional recommendations are then transformed into a revision of the revenue matrix and information structure, so as to guide the system from the “prisoner’s dilemma” pattern to a coordinated game or a quality-based competition pattern. The study further proposed a formalized diagnostic framework to describe the “inner volume trap” and formulate corresponding policies to promote more sustainable competition.

**Keywords:** Involution; Evolutionary Game Theory; Replicator Dynamics; Social Dilemma; Policy Intervention

## 1. Introduction

### 1.1 Research Background

The increasing competition in education and the labor market has given rise to the so-called “inner volume” in the Chinese context: more and more efforts are made, but the return is decreasing, accompanied by huge individual costs [1,2]. Large-scale extracurricular tutoring, strict examination training, and overtime culture (such as the widespread “996” work system) are all embodiments of a “relative advantage game” - it constantly pushes up the standard of individual effort, but does not bring corresponding overall benefits. Its dynamic mechanism is the same as the “Red Queen Effect”: you have to run faster and faster to stay still, which is reflected in the arms race in the biological world and the socio-economic championship mechanism [3]. Therefore, the “inner volume” can be regarded as a socio-economic expression of the “Red Queen Effect”.

### 1.2 Literature Review

The literature provides several theoretical frameworks to examine involution. Tournament and labor economists record “rat-race” equilibria with workers overworking to make effort commitments, yielding long hours and negative selection in upward promotions [4]. Educational research reveals “effort inflation” and incremental advances in rank with exploding study time and money [5,6]. According to these results, behavioral experiments demonstrate too much competition, even with socially inefficient contests, due to the belief that others will continue to compete [7]. Evolutionary game theory, via replicator dynamics, explains how payoff-superior strategies diffuse and become stable population shares [6]. In Prisoner’s Dilemma-like environments, defection (here, high effort) is the only evolutionarily stable outcome under standard dynamics, aligning with arms-race traps described in Red Queen analogies [3,8].

Prior work richly describes the phenomenon and static game-theoretic inefficiency, but fewer studies derive the dynamic path by which populations escalate from moderate to excessive effort or provide closed-form conditions for stability, welfare gaps, and policy levers that reshape the evolutionary vector field. This paper aims to fill these gaps by developing a transparent two-strategy replicator model. This study derives the dynamic path of escalation and provides closed-form conditions for stability, welfare losses, and the efficacy of policy levers.

### 1.3 Research Framework

Section 2 formalizes the arms-race effort game and derives the replicator dynamic. Section 3 analyzes positive

and negative impacts using explicit formulas for equilibrium, stability, convergence, and welfare. Section 4 proposes payoff-matrix and information-structure interventions (e.g., caps, evaluation reform, cooperation mechanisms) that alter the selection gradient. Section 5 concludes with significance, limitations, and extensions.

## 2. The Model: Evolutionary Dynamics of an Effort Arms Race

This study models involution as an evolutionary game where individuals adapt their strategies based on relative payoffs. Consider a large population repeatedly matched to compete on relative performance.

Each player chooses either High Effort (H) or Low Effort (L). Let  $x \in [0,1]$ , denote the population share of H. A convenient payoff parameterization capturing competitive externalities is:

$$\pi_H(x) = A - Bx, \pi_L(x) = C + Dx \quad (1)$$

Where  $A$  is the baseline gain of H when few others exert high effort;  $B > 0$  captures congestion of H;  $C$  is the baseline gain of L;  $D$  allows spillovers to L or negative externalities if  $D < 0$ .

The selection differential is:

$$\Delta(x) = \pi_H(x) - \pi_L(x) = (A - C) - (B + D)x \quad (2)$$

Let  $\bar{\pi}(x) = x\pi_H(x) + (1-x)\pi_L(x)$  be the average payoff:

$$\begin{aligned} \pi(x) &= x(A - Bx) + (1-x)(C + Dx) = \\ &C + (A - C + D)x - (B + D)x^2 \end{aligned} \quad (3)$$

Under standard replicator dynamics:

$$\dot{x} = x(1-x)[\pi_H(x) - \pi_L(x)] = x(1-x)\Delta(x) \quad (4)$$

This implies that the strategy with a higher payoff will increase its share in the population [9]. The speed of adjustment is proportional to the current variance in strategies,  $x(1-x)$ .

Fixed points satisfy  $\dot{x} = 0$ , where the interior equilibrium solves  $\Delta(x^*) = 0$ :

$$x^* = \frac{A - C}{B + D} \quad (5)$$

Local stability follows from  $f(x) := x(1-x)\Delta(x)$  and

$$\begin{aligned} f'(x) &= (1-2x)\Delta(x) + x(1-x)\Delta'(x), \\ \Delta'(x) &= -(B + D) < 0 \end{aligned} \quad (6)$$

At  $x^*$ ,  $\Delta(x^*) = 0$ , hence

$$f'(x^*) = x^*(1-x^*)\Delta'(x^*) < 0 \quad (7)$$

So  $x^*$  is locally asymptotically stable whenever it exists. Proposition (Stability of Interior Equilibrium). The interior or fixed point  $x^* = \frac{A-C}{B+D}$  is locally asymptotically stable if it exists ( $0 < x^* < 1$ ) &  $B+D > 0$ .

While the explicit solution  $x(t)$  is complex, the implicit path allows us to analyze the duration and trajectory of the escalation process. A useful implicit time path is obtained by separation of variables on standard replicator dynamics. For  $\alpha := A-C > 0, \beta := B+D > \alpha$

$$\int \frac{dx}{x(1-x)(\alpha - \beta x)} = t + C_0 \Rightarrow \frac{1}{\alpha} \ln(x) - \frac{1}{\alpha - \beta} \ln(1-x) + \frac{\beta}{\alpha - \beta} \ln(\alpha - \beta x) = t + C_1 \quad (8)$$

This integral form is sufficient for numerical calibration and phase-diagram analysis, which can vividly depict the transition path from a low-effort to a high-effort regime. This expression supports calibration and phase-diagram plotting without solving  $x(t)$  in closed form.

Two benchmark matrix games fit the first equation in this chapter and illustrate the case narrative:

Prisoner's-Dilemma Arm Race

Crowding-adjusted Tournament: choose  $(A, B, C, D)$  s.t.

$A > C$  and  $0 < \alpha < A-C < \beta = B+D$

Prisoner's-Dilemma Arms Race: This occurs when  $D < 0$  (high effort harms low-effort players) and  $A > C$  (high effort is privately beneficial). The stable equilibrium  $x^*$  is interior, but the payoff structure creates a social dilemma where universal low effort would be socially preferable, but is evolutionarily unstable.

Crowding-adjusted Tournament: This case ( $D > 0$ ) might represent a scenario where high effort generates positive knowledge spillovers. However, if the congestion effect is large enough such that  $\beta = B+D > \alpha = A-C$ , the interior equilibrium.  $x^*$  remains stable, preventing a complete convergence to universal high effort.

Thus, the model captures the two most salient observed outcomes: escalation to universal high effort ( $x \rightarrow 1$ ), which represents full-blown involution; or a stable mixed equilibrium. ( $x = x^*$ ), which represents a persistent state of partial involution within the population.

Building directly on the selection differential introduced above, it is useful to make the calculus behind the separable dynamics completely explicit before the author turns

to empirical and interpretive issues. With the replicator law  $\dot{x} = x(1-x)\Delta\pi(x)$ , any interior trajectory satisfies

$$\int_{x_0}^{x(t)} \frac{dz}{z(1-z)\Delta\pi(z)} = t - t_0 \quad (9)$$

Two benchmarking cases clarify how the payoffs just specified map into adjustment speeds. First, when the frequency effect is locally negligible and  $\Delta\pi(x) \approx c$  is approximately constant—this corresponds to  $D = B$  in the linear parameterization—then  $\dot{x} = cx(1-x)$  integrates to the logistic closed form with characteristic time  $1/|c|$ .

$$x(t) = \frac{1}{1 + \left(\frac{1}{x_0} - 1\right)e^{-c(t-t_0)}} \quad (10)$$

Second, when  $\Delta\pi$  is genuinely frequency dependent but smooth, a first-order Taylor expansion around an interior root  $x^*$  gives  $\Delta\pi(x) \approx \Delta\pi_1^-(x^*)(x-x^*)$ ; the linearized dynamics  $\dot{x} \approx \gamma(x-x^*)$  with  $\gamma := x^*(1-x^*)\Delta\pi_1^-(x^*)$  yield the exponential law

$$x(t) - x^* \approx (x_0 - x^*)e^{\gamma(t-t_0)}, t_\epsilon \approx \frac{1}{|\gamma|} \log\left(\frac{|x_0 - x^*|}{\epsilon}\right) \quad (11)$$

Both formulas tie directly to the primitives  $A, B, C, D$ : in the linear case  $\Delta\pi_1^-(x) = D - B$ ,

so more crowding  $B$  (compared to  $D$ ) makes the negative slope steeper, larger  $|\gamma|$ , and hence shortens the time for moving into a region of the interior rest point after the system is on the “correct” side of it. This is important for interpretation: policies that move the selection gradient (by reducing the weight on relative hours, or by taxing fatigue) not only move steady states; they also change convergence rates in a computable, measurable way.

For calibration and welfare accounting, it is also convenient to track the average payoff.

$$\bar{\pi}(x) = x\pi_H(x) + (1-x)\pi_L(x) \quad (12)$$

As a function of the state. Differentiating,

$$\bar{\pi}'(x) = \pi_H(x) - \pi_L(x) + x\pi_H'(x) - (1-x)\pi_L'(x) = \Delta\pi(x) + x(1-x)(\pi_H'(x) - \pi_L'(x)) \quad (13)$$

In the linear specification, where  $\pi_H - \pi_L = (A-C) + (D-B)x$ , the second term vanishes and

$\bar{\pi}'(x) = \Delta\pi(x)$ . Thus, the same crossing that determines the fixed point also maximizes average static payoff when payoffs are affine in  $x$ . When the environment deviates

from linearity (through convex fatigue or spillovers),  $\bar{\pi}_1^-(x)$  shifts accordingly; the sign and curvature of  $\bar{\pi}(x)$  around  $x^*$  Then tell us whether small policy rotations raise both efficiency and stability, foreshadowing the analysis in the next section.

### 3. Analysis

#### 3.1 Short-Run Outcomes and Functional Attributes

##### 3.1.1 Peak performance and short-run output

The most immediate and observable effect of an effort escalation is a transient boost in aggregate output. When  $\Delta(x) > 0$  for relevant  $x$ , the replicator vector field pushes  $x$  upward, temporarily raising metrics correlated with effort intensity. Define a simple performance index  $P(x)$  proportional to  $\bar{\pi}(x)$  or to H-specific outcomes:

$$P_{H(x)} := x \phi_H, \quad P_{L(x)} := (1-x)\phi_L \quad (14)$$

So the aggregate index  $P(x) = P_H(x) + P_L(x)$  rises with  $x$  if  $\phi_H \gg \phi_L$

##### 3.1.2 Tournament sorting and information revelation

Treat promotions as a constrained maximization over observed outputs  $y = e + ?$  with effort  $e \in \{e_H, e_L\}$  and nice  $?$ . Under the standard monotone likelihood ratio property, a higher  $e_H$  stochastically shifts  $y$  upward, improving the revelatory power of contests. The posterior odds of being a high type satisfy, \[ \]

$$\frac{Pr(high|y)}{Pr(low|y)} = \frac{f(y|Oe_H) Pr(high)}{f(y|Oe_L) Pr(low)} \quad (15)$$

Which increases in  $y$ ; tournament cutoffs thus select for higher types more reliably in the short run [4,7]. However, this sorting efficiency often comes at the expense of long-term welfare.

##### 3.1.3 Adaptive resilience

The evolutionary form itself might grant a functional benefit: adaptive resilience. Under repeated shocks, the replicator moves towards strategies yielding higher realized payoff; populations under strong selection gradients acquire robustness helpful in regime shifts [3,8].

A stylized benefit is measured by the speed of adaptation near  $x^*$ :

$$\left( \frac{dx}{dt} \right)_{x^*} = f'(x^*) = x^*(1-x^*)\Delta'(x^*) < 0 \quad (16)$$

So larger  $\Delta'(x^*)$  produces faster mean reversion and readiness for shocks. In essence, a population deeply entrenched in a competitive arms race is also one that is highly responsive to changing environmental conditions, for better or worse.

The global geometry of trajectories can be summarized by a Lyapunov potential. Define

$$V(x) := -\int_0^x \Delta\pi(z) dz \quad (17)$$

Along any interior solution, this study has equality only at rest points or on the boundaries.

$$\begin{aligned} \frac{d}{dt} V(x(t)) &= -\Delta\pi(x(t))\dot{x}(t) = \\ &= -x(t)(1-x(t))[\Delta\pi(x(t))]^2 \leq 0 \end{aligned} \quad (18)$$

Suppose  $\Delta\pi$  has a single simple root and preserves sign on either side (the “single-crossing” property induced by the linear A,B,C,D case, or more generally by monotone crowding). In that case,  $V$  strictly decreases along nontrivial paths, and the interior equilibrium is globally attracting on  $(0,1)$ . This potential view makes two empirical predictions: (i) shock-and-recovery paths are monotone in  $V$ , so overshooting requires non-monotone  $\Delta\pi$  or state dependence outside the model; (ii) the area under  $[\Delta\pi(x)]^2$

scaled by  $x(1-x)$  controls the speed at which “excess effort” is dissipated after a de-escalation reform.

Heterogeneity and exploration modify the same calculus without overturning its conclusions. With a small replicator-mutator term  $\mu(1-2x)$  added to the right-hand side,

the rest point shifts to  $x_\mu^* \approx x^* + \mu(1-2x^*)/[-\Delta\pi_1^-(x^*)]$

, keeping the state strictly interior and raising the effective  $|\gamma|$  near the new rest point. With two cost types  $(b_\ell, b_h)$ ,

the induced  $\Delta\pi(x)$  remains smooth and single-crossing; the interior fixed point exists under the same sign conditions, but the level of  $x^*$  is now sensitive to the cross-type margin where marginal costs of extra hours bite. In both cases, the separable form remains: time-to-threshold and convergence envelopes are still given by the integral law and its linear approximation.

To connect these dynamics to the short-run outcome subsections below, note that the very objects driving speed and direction, and the logistic constant  $c$  in the degenerate case, are also sufficient statistics for transient peaks in tournament metrics. When a policy temporarily boosts the private return  $A$  to high effort while leaving crowding  $B$

unchanged,  $\Delta\pi$  shifts upward and  $|\gamma|$  typically grows, so observed indicators spike quickly even if the long-run  $x^*$  does not move much. By contrast, policies that reduce  $B$  (e.g., de-weighting relative hours) rotate the field and can lower the peak-on-impact by shrinking  $|\gamma|$ , yet they improve both steady-state welfare and stability.

## 3.2 Long-Run Inefficiencies and Social Costs

### 3.2.1 Collective inefficiency and welfare loss

To quantify the social cost of involution, this study first derives the population share that maximizes average payoff,  $\bar{\pi}(x)$ , which defines the social optimum. The socially optimal share  $x^{opt}$  maximizes average payoff in:

$$\frac{d\bar{\pi}}{dx} = A - C + D - 2(B + D)x = 0 \Rightarrow x^{opt} = \frac{A - C + D}{2(B + D)} \quad (19)$$

Compared with the evolutionary equilibrium, the welfare gap is:

$$\Delta W = \bar{\pi}(x^*) - \max_x \bar{\pi}(x) = \bar{\pi}\left(\frac{A - C}{B + D}\right) - \bar{\pi}\left(\frac{A - C + D}{2(B + D)}\right) \leq 0 \quad (20)$$

Unless  $D = 0$  and special knife-edge equalities hold,  $x \neq x^{opt}$  with the PD-type case yielding  $x^*$  at or near 1 and  $\bar{\pi}$  strictly below the cooperative benchmark. This formalizes the Pareto-inferiority of arms-race equilibria observed empirically [2,6,7,9].

### 3.2.2 Innovations stagnation under metric myopia

A critical long-term cost of involution is the potential stifling of innovation. This occurs when the tournament rewards measurable effort intensity ( $e$ ) rather than true quality or innovation ( $Q$ ). To model this, suppose quality/innovation  $Q$  be a concave function of effective effort  $e$ , but suppose tournament metrics reward measurable intensity  $e$  instead:

$$Q(e) = \theta\sqrt{e} \quad (21)$$

While rewards depend on  $e$ . If  $e = \eta e$  with  $\eta < 1$  when everyone pursues the same tactics, marginal returns to  $Q$  shrink as  $x$  rises:

$$\frac{dQ}{de} = \frac{\theta\eta}{2\sqrt{\eta e}} \downarrow \text{ine}, \eta = \eta(x) \downarrow \quad (21)$$

So metric-driven arms races crowd out novel investment, matching observed stagnation in involved systems [5,6,9]. This creates a misallocation of resources where players

optimize for metrics rather than value creation, leading to a collective stagnation.

### 3.2.3 Human Costs: Burnout and Inequality

Beyond collective efficiency, involution imposes significant human costs in the form of burnout and exacerbated inequality. Model fatigue as a convex cost  $c(e) = e^2\kappa/2$  borne by H-players, with net payoff  $\pi_H(x) - c(e_H)$ .

As  $x \uparrow$  reduces  $\pi_H$ , but  $e_H$  remains high under tournament pressure, the net surplus erodes:

$$\Pi_H(x) = (A - Bx) - \frac{\kappa e_H^2}{2} \quad (22)$$

For heterogeneous budgets  $b$ , feasible effort satisfies.

$$e_H \leq \sqrt{\frac{2b}{\kappa}}, \text{ so wealthier agents sustain higher } e_H \text{ and}$$

outlast poorer agents; hence inequality widens over time, consistent with evidence on tutoring gaps and overtime survivorship [1,3,9]. Thus, the arms race not only reduces average welfare but also leads to a more unequal distribution of it, as those with greater initial resources are better equipped to endure the competition.

### 3.2.4 Strategic Trap and Cultural Spillovers

Perhaps the most pernicious aspect of involution is that it constitutes a strategic trap. The basin of attraction of high-effort norms can be large. In PD-type games, any  $x > x$  flows to  $x = 1$ . A simple Lyapunov function proving

monotone drift toward  $x = 1$  uses  $V(x) = \ln \frac{x}{1-x}$  with

$$\dot{V} = \Delta(x), \text{ which is positive for } x < \frac{1}{2}, \text{ locking society}$$

into an escalating culture that corrodes trust and cooperation [3,7,8].

## 4. Policy Interventions: Reshaping Payoff Structures and Beliefs

Building upon the analysis of involution's costs and dynamics in Section 3, this section proposes two fundamental intervention pathways. The first aims to redesign the game itself by altering the parameters  $(A, B, C, D)$  of the incentive structure (Section 4.1). The second seeks to influence the game's play by modifying the perceptions and behavioral patterns of the participants (Section 4.2). Both types of interventions target the selection gradient  $\Delta(x)$ , aiming to steer the system from a Prisoner's Dilemma regime towards coordination or quality-based competition.



#### 4.1 Structural Interventions: Altering the Pay-off Matrix

Interventions should alter  $\Delta(x)$  so that the selection gradient does not push toward excessive  $x$ .

(i) Imposing Regulatory Boundaries: Caps and Floors. Impose caps on measurable intensity (homework hours, overtime) and floors on harmful tactics (predatory pricing). This effectively reduces  $B$  (less crowding from  $H$ ) and raises  $C$  (baseline payoff of  $L$ ), shrinking  $\alpha = A - C$  and enlarging  $\beta = B + D$  to move  $x^*$  left:  $x^* = \frac{A-C}{B+D} \downarrow$

if  $A - C \downarrow$  or  $B + D \uparrow$ . Real-world examples include the Double-Reduction policy limiting cram schooling and homework, and legal rulings against 996 schedules [1,2,10].

(ii) Metric Reform (Quality over Quantity). Change evaluation rules so rewards depend on quality/innovation  $Q$  rather than raw hours. If bonuses weight  $Q$  with a coefficient  $\lambda$ , and hours with  $1 - \lambda$ , then  $\pi_H$  becomes

$$\pi_H^{new}(x) = \lambda Q(x_H) + (1 - \lambda)(A - Bx) \quad (23)$$

with  $\frac{\partial \pi_H^{new}}{\partial x}$  less negative when  $\lambda$  is large and  $Q$  depends

less on others' effort, thereby flattening  $\Delta'(x)$  and reducing escalation [6,7,9].

#### 4.2 Informational and Behavioral Interventions: Managing Beliefs and Costs

(i) Soft Coordination (Repeated-Game Devices). With communication and repeated interactions, introduce credible mutual restraint (e.g., homework conventions, industry codes). In a stage game with payoffs  $(\pi_L, \pi_H)$  And discount factor  $\delta$ , trigger strategies sustain cooperative  $L$  if

$$(1 - \delta)[\pi_L(0) - \pi_H(0)] \geq \delta \quad (24)$$

This converts a PD into a coordination game in repeated play, shrinking the attraction to high  $x$  [3,8].

(ii) Internalizing Fatigue Costs. Let institutions price fatigue with credits/penalties  $t(e)$ , so the net payoff is  $\pi_i(x) - t(e)$ . Choosing

$$t(e) = \tau e^2 (\tau > 0) \quad (25)$$

makes extreme effort privately costly, lowering  $\alpha$  and shifting  $x^*$  left. Health safeguards, mandated rest, and overtime pay are practical instruments [1,10].

(iii) Establishing Opportunity Floors through Public Provision. Subsidize baseline resources to raise  $C$  (public education, mentoring).

$$\frac{\partial x^*}{\partial C} = -\frac{1}{B+D} < 0 \quad (26)$$

So increasing  $C$  directly reduces equilibrium high-effort prevalence and narrows inequality [1,2, 6].

(iv) Belief Management and Transparency. Since excessive competition is driven by beliefs that “others will compete anyway”, public signals can reduce perceived  $x$  or  $\Delta(x)$ : cohort-wide commitments, transparent policy enforcement, and norms that de-glorify all-nighters shift expectations. In Bayesian terms, posterior  $E[x | \text{signal}]$  falls, reducing  $x(1-x)\Delta(x)[7]$ .

Briefly, structural interventions provide the institutional foundation for intervention reduction, and informational and behavioral interventions, by affecting individual decision-making environments and beliefs, can strongly augment the efficacy of structural measures or assume a primary role where their direct implementation is challenging. Together, they form a complementary system redesign toolkit.

#### 4.3 A Dynamic Policy Optimization Framework

A dynamic optimization perspective is instructive in integrating the aforementioned interventions into a cohesive policy evaluation framework.

A calculus-of-variations lens makes these policy trade-offs transparent. Let  $\theta$  parameterize an institutional lever. Let the discounted welfare be

$$J(\theta) = \int_0^\infty e^{-\rho t} W(x_\theta(t)) dt, \rho > 0 \quad (27)$$

where  $x_\theta$  solves  $\dot{x} = x(1-x)\Delta\pi(x; \theta)$ . A small permanent reform  $\theta \mapsto \theta + \delta\theta$  induces a path variation  $\delta x$  satisfying

$$\delta \dot{x} = x_\theta(1-x_\theta)\partial_x \Delta\pi(x_\theta; \theta) \Gamma(t) \delta x + x_\theta(1-x_\theta)\partial_\theta \Delta\pi(x_\theta; \theta) \Psi(t) \delta\theta \quad (28)$$

Variation of constants yields

$$\delta x(t) = \delta\theta \int_0^t \exp\left(-\int_s^t \Gamma(u) du\right) \Psi(s) ds \quad (29)$$

and hence

$$\begin{aligned} \frac{dJ}{d\theta} &= \int_0^\infty e^{-\rho t} W'(x_\theta(t)) \delta x(t) dt = \\ &= \delta\theta \int_0^\infty \int_0^t e^{-\rho t} W'(x_\theta(t)) \exp\left(-\int_s^t \Gamma(u) du\right) \Psi(s) ds dt \end{aligned} \quad (30)$$

If  $\theta$  reduces the “hours gradient” so that  $\partial_\theta \Delta\pi < 0$  in the relevant range, the kernel in the double integral is non-negative near the interior rest point. Thus

$\frac{dJ}{d\theta} < 0$  for  $\theta = \beta$ : welfare increases as crowding pressure is relaxed.

The framework emphasizes that effective policies must consider both steady-state and transitional paths to ensure that the dynamic evolution of the system under intervention can maximize social welfare.

## 5. Conclusion

The paper formalizes the phenomenon of “arms competition-style efforts” into a simple dual-strategy evolution model. The modeling process brings three key insights. First, under replication dynamics, the system either converges to a dominant high-effort attractor (similar to the prisoner’s dilemma) or converges to a mixed equilibrium of internal stability. Second, the model can derive a closed solution of the selection gradient and equilibrium share  $x^*$ ; Third, this framework is for further study of local stability. Sex, comparative statics, and the optimal share of welfare have laid the foundation, local stability, comparative statics, and the welfare-optimal share  $x^{opt}$ . The

gap  $\left| \bar{\pi}(x^*) - \bar{\pi}(x^{opt}) \right| \geq 0$  quantifies collective inefficiency.

These outcomes are consistent with empirical patterns of schooling and labor markets: short-term gains and selection advantages exist alongside innovation stagnation, burnout, and inequality. Second, aside from positive analysis, the model offers a normative agenda for policy design. The potential interventions—structural (changing payoff parameters  $(A, B, C, D)$ ) and informational (manipulating beliefs)—are explicitly tied to adjusting the selection gradient  $\Delta(x)$ . This creates a direct causal link between policy levers and evolutionary outcomes, nudging the system out of Pareto-inferior equilibria. Finally, the analysis bridges widely observed phenomena with a rigorous dynamic model, providing a diagnostic toolkit for policymakers: identify parameters  $(A, B, C, D)$ , measure  $x$ , compute  $x^*$  vs  $x^{opt}$ , and act on levers that reduce the selection for wasteful escalation.

While the parsimonious model offers clarity and tractability, it also points to natural avenues for extending the research frontier. However, the model is binary and deterministic. Important extensions include continuous effort  $e \in [0, \bar{e}]$  with cost  $c(e)$  and contests  $p(e_i, e_j)$

; replicator–mutator dynamics with exploration rate  $\mu$ :  $\dot{x} = x(1-x)\Delta(x) + \mu(1-2x)$ , which prevents absorbing corners; and stochastic shocks to payoffs and beliefs. Empirical calibration with panel data on workloads, outcomes, and policy changes would further validate the levers. Ultimately, by mapping the dynamic logic of involution, this framework provides not just a diagnostic for a pervasive social problem, but also a generative toolkit for designing more sustainable and efficient competitive ecosystems.

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